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The Minimum Test Collection Problem

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Abstract

In this paper we consider an approach to solve the minimum test collection problem. This approach is based on an explicit reduction from the problem to the satisfiability problem.

Keywords: minimum test collection problem, satisfiability, **NP**-complete

Investigation of different regularities plays an important role for the detection of various knowledge (see e.g. [1] – [8]). In particular, the minimum test collection problem can be used for rapid identification of unknown pathogens (see e.g. [9]). Let

$$D = \{D[1], \ldots, D[n]\},$$

$$T = \{T[1], \ldots, T[m] \mid T[i] \subseteq D\}.$$

The minimum test collection problem (MTC):
Instance: *Given a set D and a positive integer d.*
Question: *Is there a set*

$$S \subseteq T$$

such that $|S| \leq d$ and for any $1 \leq i < j \leq n$, there is $T[k] \in S$ such that

$$|\{D[i], D[j]\} \cap T[k]| = 1?$$

Note that MTC is **NP**-complete (see e.g. [10]). Encoding hard problems as instances of SAT and solving them with different efficient satisfiability algorithms has caused considerable interest (see e.g. [11] – [15]). In this paper, we consider an approach to solve the MTC problem. Our approach is based on an explicit reduction from the problem to the satisfiability problem. Let

$$\varphi[1] = \wedge_{1\le i\le d} \vee_{1\le j\le m} x[i,j],$$

$$\varphi[2] = \wedge_{1\le i\le d} \wedge_{1\le j[1]<j[2]\le m} (\neg x[i,j[1]] \vee \neg x[i,j[2]]),$$

$$\varphi[3] = \wedge_{1\le i\le d} \wedge_{1\le j\le m} \wedge_{1\le k\le n, D[k]\in T[j]}(\neg x[i,j] \vee y[i,k],$$

$$\varphi[4] = \wedge_{1\le i<j\le n} \vee_{1\le k\le d} z[i,j,k],$$

$$\varphi[5] = \wedge_{\substack{1\le i<j\le n\\ 1\le k\le d}}((\neg z[i,j,k] \vee y[k,i] \vee y[k,j]) \wedge (\neg z[i,j,k] \vee \neg y[k,i] \vee \neg y[k,j])).$$

Let $\xi = \wedge_{i=1}^{5}\varphi[i]$. It is easy to check that there is a set $S \subseteq T$ such that $|S| \le d$ and for any $1 \le i < j \le n$, there is $T[k] \in S$ such that $|\{D[i], D[j]\} \cap T[k]| = 1$ if and only if ξ is satisfiable. It is clear that ξ is a CNF. So, ξ gives us an explicit reduction from MTC to SAT. Now, using standard transformations (see e.g. [16]) we can obtain an explicit transformation ξ into ζ such that $\xi \Leftrightarrow \zeta$ and ζ is a 3-CNF. Clearly, ζ gives us an explicit reduction from MTC to 3SAT. In papers [17, 18] the authors considered some satisfiability algorithms. Our computational experiments have shown that these algorithms can be used to solve MTC.

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